WORK, POWER & ENERGY

① If a_1 and a_2 two non collinear unit vectors and. if $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$, then the value of $(\vec{a}_1 - \vec{a}_2)\cdot(2\vec{a}_1 + \vec{a}_2)$ is (key $= \frac{1}{2}$)

② A block of mass $m = 1$ kg moving on a horizontal Surface with speed $v_i = 2 ms^{-1}$ enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely Proportional to x over this range. $F_r = \frac{-k}{x}$ for $0.1 < x < 2.01$ m $= 0$ for $x < 0.1$ m and $x > 2.01$ m where $k = 0.5$ J. what is the final kinetic energy and speed v_f of the block as it Crosses this patch? (key $1 ms^{-1}$)

③ A body of mass 50 kg starts from rest and slides down a smooth inclined plane. find the force "p" applied at an angle 30° to the inclined plane so that the body attains a velocity of $10 ms^{-1}$ after sliding a distance of 10 m. (key $\therefore P = 129.5$ N).

④ Under the action of force, 2 kg body moves such that its position x as a function of time t is given by $x = \frac{t^3}{3}$ x is in metre and t in second. Calculate

(5) A lorry and a car moving with the same kinetic energy are brought to rest by the application of brakes, which provide equal retarding forces. Which of them will come to rest in shorter time? Which will come to rest in less distance (a) key - they came to rest after travelling the same distance) b) (key - the car comes to rest earlier)

(6) Consider "n" number of indentical cubes each of mass "m" and side "a" lying on a level ground. Find the work done in piling them one above the other. key $\left[w = mg\frac{a}{2}n(n-1)\right]$

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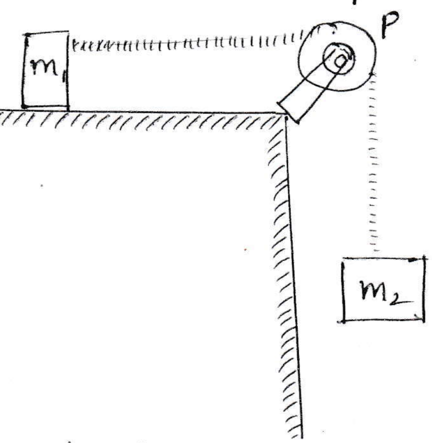
(7) A particle of mass 'm' is moving in a horizontal circle of radius r, under a centripetal force equal to $-(k/r^2)$, where k is constant. What is the total energy of the particle? key $\left[E = -\frac{k}{2r}\right]$

(8) A uniform chain of length 'L' and mass 'm' is on a smooth horizontal table, with $\frac{1}{n}$th part of its length hanging from the edge of the table. Find the kinetic energy of the chain as it completely slips off the table. key $\left[K_f = \frac{mgL}{2}\left[1 - \frac{1}{n^2}\right]\right]$

(9) Two blocks are connected by a string as shown in figure. They are released from rest. Show that after they moved a distance L, their common speed is given by $\sqrt{\frac{2(m_2-\mu m_1)gL}{(m_1+m_2)}}$ where μ is the coefficient of friction (key - $v = \sqrt{\frac{2(m_2-\mu m_1)gL}{(m_1+m_2)}}$)



(10) A boy throws a ball with initial velocity u at an angle of projection θ from a tower of height H. Neglecting air resistance, find (a) how high above the building the ball rise, and (b) its speed just before it hits the ground. (key a) $\left(h = \frac{u^2 - u^2\cos^2\theta}{2g}\right)$ b $\left(v = \sqrt{u^2+2gH}\right)$

(11) A nail is located below the point of suspension of a simple pendulum of length 'l'. The bob is released from horizontal position. If the bob loops a verticle circle with nail as centre, (1) find the distance of nail from point of suspension (2). find the angle 'θ' with the lower vertical at which the resultant acceleration of the bob is along the horizontal, when the pendulum is released from horizontal position. (key - $x = 3l/5$ (a)

(b) $\theta = \tan^{-1}(\sqrt{2})$

(12) A mass m is released from the top of a vertical circular track of radius r with a horizontal speed v_0. Calculate the angle θ with respect to the vertical where it leaves contact with the track. [key $\theta = \cos^{-1}\left[\frac{v_0^2}{3rg} + \frac{2}{3}\right]$]

(13) Consider a pendulum consisting of a massless string with a mass at one end. The mass is held with the string horizontal and then released. The mass swings down, and on its way back up, the string is cut at point P when it makes an angle of with θ the vertical. Find the angle for θ which horizontal range 'R' is maximum.

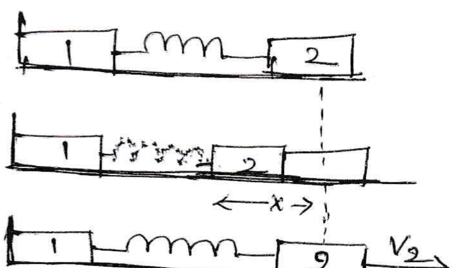
[key ($\tan\theta = 1/\sqrt{2}$)]

(14) A pendulum bob of mass 'm' is held out in the horizontal position and then released from rest. If the string is of length l (1) what is the velocity of the bob and tension in the string when the bob reaches the lowest position.

(2) If the breaking strength of the string is 'nmg'. find the maximum angle 'θ' with respect to the vertical at which the bob is to be released to avoid breaking of the string. [key (1) $v = \sqrt{2gl}$.), $T = 3mg$.]

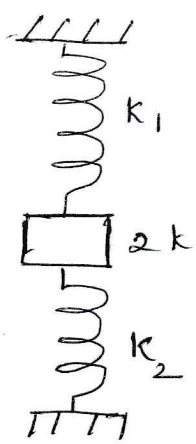
(2) $\cos\theta = 3-n$

(15) Two bars of masses 'm_1' and 'm_2' connected by a weightless spring of stiffness constant k as shown in figure, rest on a smooth horizontal plane. Bar 2 is shifted through a small distance x to the left and then released. find the velocity of the centre of mass of the system after bar 1 breaks off the wall.



key $\left[v_c = x\sqrt{\dfrac{km_2}{(m_1+m_2)}} \right]$

(16) 2 kg block is connected with two springs of force constants $k_1 = 100$ N/m and $k_2 = 300$ N/m as shown in figure. The block is released from rest with the springs unstretched. find the accelerations of the block in its lowest position ($g = 10$ m/s^2)



key – (10 m/s^2 upwards)

(17) Consider a simulation with a car of mass 1000 kg moving with a speed 18.0 km/h on a road and colliding with a horizontally mounted spring of spring constant 6.25×10^3 N m^{-1}. Taking

the maximum compression of the spring? (Key: $x_m = 1.35m$)

(18) An electric pump on the ground floor of a building taken 15 minutes to fill a tank of volume $30 m^3$ with water. If the tank is 40 m above the ground and the efficiency of the pump is 30%, find the electric power consumed by the pump in filling the tank. [density of water $= 10^3 kg/m^3$] [Key 43.56 kw]

(19) A constant power 'p' is applied to a particle of mass 'm'. The displacement of the particle when its velocity increases from v_1 to v_2 (ignore friction)

key: $\left[s = \frac{m}{3p}(v_2^3 - v_1^3) \right]$

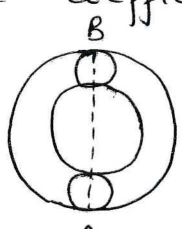
(20) A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration n.a is varying with time t as $a_c = k^2 r t^2$ where k is a constant. What is the power delivered to the particle by the forces acting on it? [key $= mk^2r^2t$]

(21) A ball of mass m hits a floor with a speed v making an angle of incidence θ with the normal. The coefficient of restitution is 'e'. Find the speed of the reflected ball and the angle of reflection of the ball.

(22) A ball of mass m collides with the ground at an angle α with the vertical. If the collision lasts for time t, the average force exerted by the ground on the ball is : (e = coefficient of restitution between the ball and the ground) [key - $F = \frac{mu \cos\alpha (1+e)}{t}$]

(23) A ball strikes a horizontal floor at an angle $\theta = 45^\circ$ with the normal to floor. The coefficient of restitution between the ball and the floor is $e = 1/2$, the fraction of its kinetic energy lost in collision is (key - $3/8$)

(24) Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. At time t = 0, A is projected along the groove and it first impinges on B at time $t = T_1$ and again at time $t = T_2$. If 'e' is the coefficient of restitution, find the ratio of T_2/T_1



(25) After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the combined particle becomes half the initial speed of either particle. The angle between

(26) In a nuclear reactor a neutron of high speed (typically $10^7\ ms^{-1}$) must be slowed to $10^3\ ms^{-1}$ so that it can have a high probability of interacting with isotope $^{235}_{92}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nuclei like deuterium (or) Carbon which has a mass of only a few times the neutron mass.

(27) Prove that in case of oblique elastic collision of two Particles of equal mass if one is at rest, the recoiling Particles always move off at right angles to each other?

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

(1) When 'n' number of particles each of mass 'm' are at distances $x_1 = a$, $x_2 = ar$, $x_3 = ar^2$ $= x_n = ar^n$ units from origin on the x-axis, then find the distance of their Centre of mass from origin. k $\left[= \frac{a(1-r^n)}{n(1-r)} \right]$.

(2) Two blocks of masses 10 kg and 30 kg are placed on x-axis. The first mass is moved on the axis by a distance of 2 cm right. By what distance should the Second mass be moved to keep the position of Centre of mass unchanged. key [Therefore the second block should be moved left through a distance of $\frac{2}{3}$ cm to keep the Position of Centre of mass unchanged].

(3) A 1 m long rod having a Constant cross sectional area is made of four materials. The first 0.2 m are made of iron, the next 0.3 m of lead, the following 0.2m of aluminium and the remaining part is made of copper. Find the centre of mass of the rod. The densities of iron, lead, aluminium and copper are 7.9×10^3 kg/m³, 11.4×10^3 kg/m³, 2.7×10^3 kg/m³ and 8.9×10^3 kg/m³ respectively key [x_c = 0.481m from the end "O" of the

(11) If the linear density of a rod of length L varies as $\lambda = A + Bx$, find the position of its centre of mass.

Key $= \dfrac{L(3A + 2BL)}{3(2A + BL)}$

(12) If the linear density of a rod of length L varies as $\lambda = \dfrac{kx^2}{L}$ where k is a Constant and x is the distance of any point from one end, then find the distance of Centre of mass from the end at $x = 0$. (key - $\dfrac{3L}{4}$)

(13) A roller of mass 300 kg rests against a step of height 20 cm. If the radius of the roller is 50 cm find the force to be applied passing through its Center of mass to take the roller on to the step. Key - 3920 N

(14) A particle is projected at time $t = 0$ from a point 'O' with a speed 'v' at an angle 'θ' to horizontal. Find the torque of gravitational force on projectile about the origin at time 't'. (xy plane is vertical plane). key - $\bar{\tau} = -mg\, u\cos\theta\, t\, (\hat{k})$

(15) A force F is applied on the top of a Cube as shown in figure. The Coefficient of friction between the cube and the ground is μ. If F is gradually increased. find the value of μ for which the ...

(16) A boy and a man Carey a uniform pole of length 8 m and of mass 60 kg by supporting it on their shoulders. They are located at the ends of the pole. Where should a load of 90 kg be suspended from boy's end so that boy carries only one-third of the total load? (key $x = 6.22$ m).

(17) A uniform rod of length L is held vertically on a horizontal floor fixing its lower end, the rod is allowed to fall onto the ground. Find (i) its angular velocity at that instant of reaching the ground (ii) The linear velocity with which the tip of rod falls to floor.

[Key i) $\omega = \sqrt{\frac{3g}{L}}$ ii) $= \sqrt{3gL}$]

(18) The angular frequency of a fan of moment of inertia 0.1 kg m^2 is increased from 30 rpm to 60 rpm when a torque of 0.03 Nm acts on it. Find the number revolutions made by the fan while the angular frequency is increased from 30 rpm to 60 rpm.

[key - 7.855 rev].

(19) From a disc of mass 'm' and radius 'r', a circular Portion of radius $\frac{r}{3}$ is removed from the edges. Find the M. I of the ...

normal through Centre of original disc (key $I_2 = \frac{4mr^2}{9}$)

(20) A disc of mass m and radius R has a Concentric hole of radius r. find the M.I about a normal axis to its plane and passing through Centre of disc.

key $I = \frac{m}{2}(R^2+r^2)$

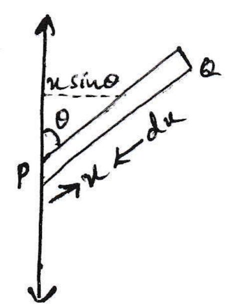
(21) Particles of masses 1g, 2g, 3g, ... 100g, are kept at the marks 1 cm, 2 cm, 3 cm, ... 100 cm respectively on a meter scale. Find the moment of inertia of the system of particles about a perpendicular bisector of the meter Scale. (key- 0.43 kgm^2)

(22) Four thin Uniform rods each of mass m and length l are arranged to form a square. find the moment of inertia of the system about an axis (i) Passing through its Centre and perpendicular to its plane. (ii) Passing through one of its sides, (iii) passing through a Corner and perpendicular to its plane. (iv) About a diagonal of the system. [key i) $\frac{4ml^2}{12} \times 4$ ii) $\frac{5}{3}ml^2$ (iii) $\frac{10}{3}ml^2$ (iv) $\frac{2}{3}ml^2$]

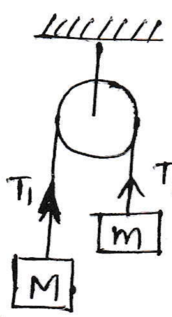
(23) Four spheres each of diameter 2a and mass m are placed with their centres on the four corners of a square of the side b. Calculate the moment of inertia of the system about any side of the square.

[key $I = \frac{8}{5} ma^2 + 2mb^2$].

(24) A rod PQ of mass 'm' and length L is rotated about an axis through 'P' as shown in figure. Find the moment of inertia of the rod about the axis of rotation. [key. $I = \frac{mL^2}{3} . \sin^2 \theta$]

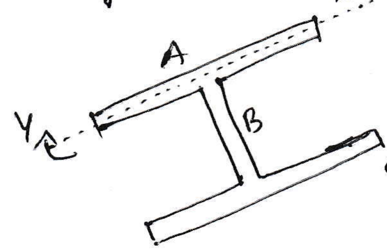


(25) The pulley of Atwood's machine has a moment of inertia 'I' about its axis and its radius is R. Find the magnitude of acceleration of the two blocks assuming the string is light and does not slip on the pulley. [key $a = \frac{(M-m)gR^2}{I + (M+m)R^2}$]

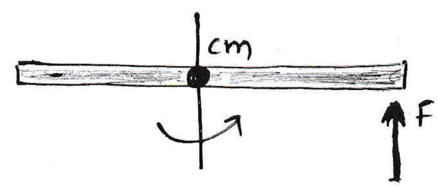


(26) A rigid body is made of three identical thin rods, each of length 'L' fastended together in the form of the letter 'H'. The body is free to rotate about

one of the legs of 'H'. The body is allowed to fall from rest from a position in which the plane of 'H' is horizontal. What is the angular speed of the body when the plane of 'H' is vertical? [Key $\omega = \frac{3}{2}\sqrt{\frac{g}{L}}$].



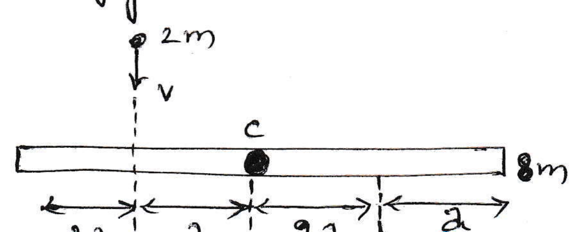
(27) A uniform rod of mass 'm' and length 'l' is on the smooth horizontal surface. When a constant force 'F' is applied at one end of the rod for a small time 't' as shown in the figure. Find the angular velocity of the rod about this centre of mass.



[Key ⇒ $\omega = \frac{6Ft}{ml}$]

(28) A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v and v respectively, strike the bar (as shown in fig) and stick to the bar after collision. Calculate (a) velocity of the centre of mass (b) angular velocity about centre of mass and (c) total kinetic energy, just after collision.

Key (a) $v = 0$ (b) $\omega = (v/5a)$
(c) $\frac{3}{5}mv^2$



(29) A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad s^{-1}, which of the two will starts to roll earlier? The co-efficient of kinetic friction is $\mu_k = 0.2$. (key - The disc begin to roll earlier than the ring for the same values of r and ω_0)

(30) A cylinder of mass 5 kg and radius 30 cm, free to rotate about its axis, receives an angular impulse of 3 kg ms^{-1} initially followed by a similar impulse after every 4 s. What is the angular speed of the cylinder 30 s after the initial impulse if the cylinder is at rest initially? (key $\omega_2 = 106.7$ rad s^{-1})